# British Mathematical Olympiad

**Round 1 : Wednesday, 15 January 1997**

**Time allowed** *Three and a half hours.*

**Instructions**

- *Full written solutions - not just answers - are required, with complete proofs of any assertions you may make. Marks awarded will depend on the clarity of your mathematical presentation. Work in rough first, and then draft your final version carefully before writing up your best attempt. Do not hand in rough work.*
- *One* complete *solution will gain far more credit than several unfinished attempts. It is more important to complete a small number of questions than to try all five problems.*
- *Each question carries 10 marks.*
- *The use of rulers and compasses is allowed, but calculators and protractors are forbidden.*
- *Start each question on a fresh sheet of paper. Write on one side of the paper only. On each sheet of working write the number of the question in the top* left *hand corner and your name, initials and school in the top* right *hand corner.*
- *Complete the cover sheet provided and attach it to the front of your script, followed by the questions 1,2,3,4,5 in order.*
- *Staple all the pages neatly together in the top* left *hand corner.*

Do not turn over until **told to do so.**

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1. $N$ is a four-digit integer, not ending in zero, and $R(N)$ is the four-digit integer obtained by reversing the digits of $N$; for example, $R(3275) = 5723$.
   Determine all such integers $N$ for which $R(N) = 4N + 3$.

2. For positive integers $n$, the sequence $a_1, a_2, a_3, \ldots, a_n, \ldots$ is defined by
   $$a_1 = 1; \quad a_n = \left(\frac{n+1}{n-1}\right)(a_1 + a_2 + a_3 + \cdots + a_{n-1}), \quad n > 1.$$
   Determine the value of $a_{1997}$.

3. The Dwarfs in the Land-under-the-Mountain have just adopted a completely decimal currency system based on the *Pippin*, with gold coins to the value of 1 *Pippin*, 10 *Pippins*, 100 *Pippins* and 1000 *Pippins*.
   In how many ways is it possible for a Dwarf to pay, in exact coinage, a bill of 1997 *Pippins*?

4. Let $ABCD$ be a convex quadrilateral. The midpoints of $AB$, $BC$, $CD$ and $DA$ are $P$, $Q$, $R$ and $S$, respectively. Given that the quadrilateral $PQRS$ has area 1, prove that the area of the quadrilateral $ABCD$ is 2.

5. Let $x$, $y$ and $z$ be positive real numbers.
   (i) If $x + y + z \geq 3$, is it necessarily true that $\dfrac{1}{x} + \dfrac{1}{y} + \dfrac{1}{z} \leq 3$ ?
   (ii) If $x + y + z \leq 3$, is it necessarily true that $\dfrac{1}{x} + \dfrac{1}{y} + \dfrac{1}{z} \geq 3$ ?